NEW TYPE OF DAM

F. G. BAUM & COMPANY
SAN FRANCISCO, CALIFORNIA

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CHRONICLE BUILDING
SAN FRANCISCO, CAL.

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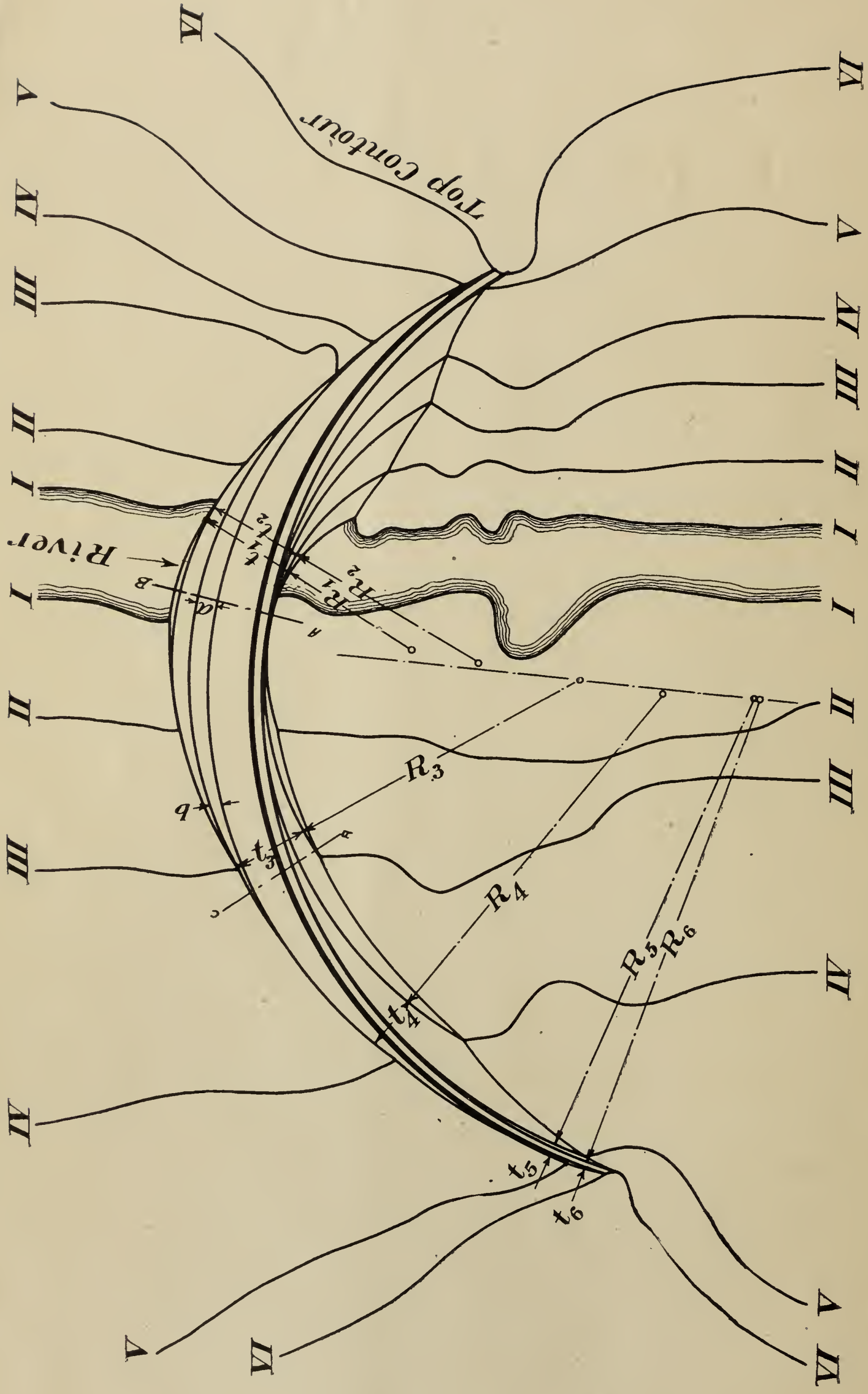

Top contour
River
I
II
III
IV
V
VI
R_1
R_2
R_3
R_4
R_5
R_6
t_1
t_2
t_3
t_4
t_5
t_6

Fig. 1

NEW TYPE OF ARCH DAM

The service which a dam of any type is expected to perform is that of keeping water back, storing it for future use, or raising its level to increase its potential energy. All types do this equally well. The best type of dam to select for any given locality will, therefore, be the one which does this in the safest and cheapest manner. The type described below has these features, when constructed where ordinary arched dams are possible of application; that is, in comparatively narrow canyons where the bottom and sides are solid rock. In the design only well known and thoroughly practical principles have been employed. These have, however, been combined in a new manner which has brought theory and practice as close to each other as is possible in any structure, thereby effecting a great saving of material. The main feature of this new type, and the one which makes the high economy of construction feasible, is the keeping of the subtended angle of the arch as nearly constant as possible, and as near to the most economical value at any elevation, as the contour of the dam site will permit. This necessitates the abandonment of the constant up-stream radius generally employed in arch dams as at present constructed; and the substitution of radii of varying length determined by the width of the canyon at any elevation. It will be shown that it is of the highest importance that the angle enclosed by the arch dam be held as nearly constant as possible, at all elevations, the length of the radius of the arch being fixed in accordance with this condition.

THE DESIGN OF THE DAM

In designing the dam it is convenient to start with the assumption which is usually made that any unit horizontal element of a curved dam, such as that for example shown in Figure 2, is a portion of a cylindrical ring, in which the average stress **q**, expressed in pounds per square foot, is equal to the radial load P (in pounds per square foot), multiplied by the radius of the exterior face and divided by the area of the section; as per equation 1:—

$$(1) \qquad q = \frac{P(R+t)}{A}$$

in which **t** equals the thickness of the dam at any given point, R is the radius of the down-stream face and A, the area of the section.

Further, the volume of concrete in any arch dam is equal to the area of the dam section, times the mean radius, times the enclosed angle.

(2) $$V = A \times R_m \times 2\theta$$

V being the volume, R_m the mean radius, 2θ the enclosed angle indicated in figure 2.

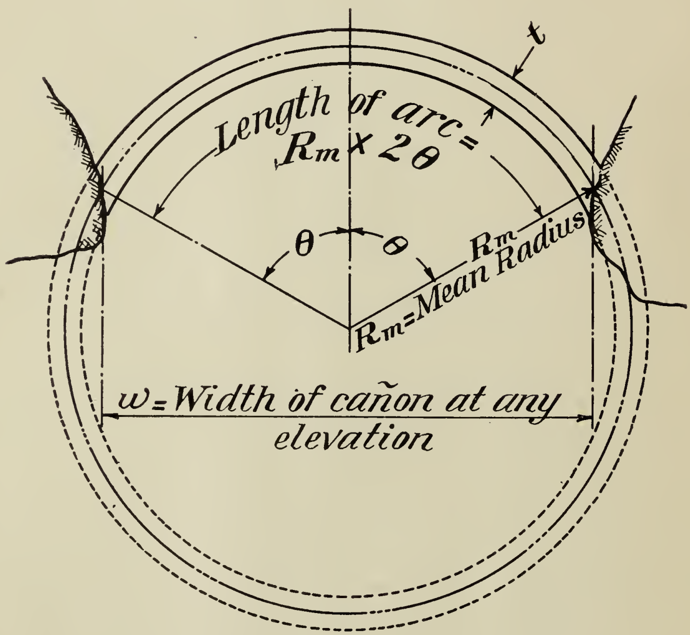


Fig. 2

The mean radius R_m, equals the radius of the down-stream face plus half the thickness **t**; and also equals half the width W, of the canyon divided by the sine of half the subtended angle.

(3) $$R_m = R + \frac{t}{2} = \frac{\frac{W}{2}}{\sin.\theta}$$

Now as the area of the dam section varies with the mean radius; the volume of masonry is:—

(4) $$V = \frac{C' \times \left(\frac{W}{2}\right)^2 \times 2\theta}{\sin.^2\theta} = \frac{K \times \theta}{\sin.^2\theta}$$

in which C' and K are constants, the latter depending upon the width of the canyon.

From equation 4 it is seen that the volume varies with

$$\frac{\theta}{\sin^2\theta}$$

The relative values of this term are graphically shown by the curve 3, in which the various angles representing 2θ constitute the abscissas and the values of

$$\frac{\theta}{\sin^2\theta}$$

constitute the ordinates; the latter for reasons above set forth being proportional to the volume of the masonry. From this curve it will be seen that the amount of masonry required for any curved or arched dam will be a minimum when the mean radius at any elevation is so chosen that the enclosed angle 2θ is about 133°; and the curve also shows that the variation in the amount of masonry required for a given dam will only be about one per cent, provided that the values of 2θ be held between the limits of 120° and 146°.

The method of dam design herein disclosed involves the independent determination of the dimensions of successive arch-shaped slices of the dam lying between predetermined levels; each slice being considered primarily as an independent structure; such slices being thereafter superposed to form the dam body.

Referring now to Figure 1, the procedure for determining the contour of the faces of the dam therein shown is as follows: A topographical survey of the canyon or valley should first be made and the contour lines should be plotted as indicated in Figure 1. In the present instance six elevations or levels have been established, forming respectively the contour lines I, II, III, IV, V and VI, the contour line VI corresponding to the level of the water retained by the dam when the latter is completed, while the coutour line I--I corresponds to the lowest dam level in the gorge or valley; the remaining lines being those of intermediate levels.

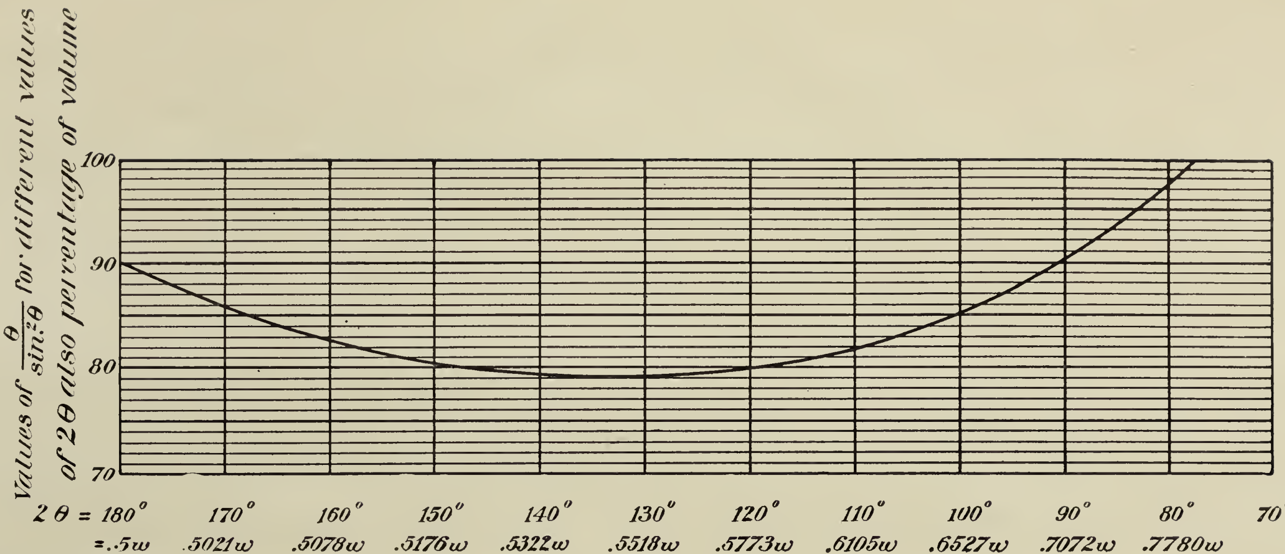
The distance across the canyon W_6 at the top (contour VI) of the dam is ascertained. Then the mean radius,

$$R_6 + \frac{t_6}{2}$$

which will give the dam the greatest strength with the least volume of material is found by means of figure 3. At this particular elevation (VI) the most economical mean radius will be

$$\frac{\frac{W_6}{2}}{\sin\theta}$$

At the top of the dam it will generally be found of advantage to choose 2θ near the upper limit (146°) for greatest economy, and at the bottom to correspondingly choose 2θ near the lower limit (120°). After the most economical mean radius for this elevation has been ascertained, the thickness t_6 may be algebraically determined, from the foregoing equations.



Radius = Constant x w

Fig. 3

Volume of concrete in any arch dam = Area of section x mean radius x enclosed angle

$$\text{Mean radius} = \frac{\text{half the width of cañon}}{\text{sine half the enclosed angle}} = \frac{\frac{w}{2}}{\text{sine}\ \theta}\ ;\ \text{Area} = \text{Constant} \times \text{radius}.$$

$$\text{Volume} = C\frac{\left(\frac{w}{2}\right)^2}{\text{sine}^2\theta} \times 2\theta = K\frac{\theta}{\text{sine}^2\theta}$$

In curve Fig. 3 the values of the term $\frac{\theta}{\text{sine}^2\theta}$ *for different values of* 2θ *are given showing that the volume of masonry is a minimum when the enclosed angle* (2θ) *is about* 133° *and also showing that for* 2θ *varying between* 120° *and* 146° *the variations in volume of masonry is only about 1 per cent.*

The first relatively abrupt change of slope of the canyon sides occurs at elevation V. The distance across the canyon at this elevation being measured, the most economical mean radius of the section is found in the same manner as the mean radius for Elevation VI was determined. After the mean radius has been settled upon, the thickness t_5 is computed and the up and down stream radii are determined. The center of curvature of the arch slice at elevation V with a thickness t_5 does not necessarily lie on the same center line as the center of curvature of the arch slice at elevation VI; in fact it practically never will do so, as perfectly even slopes of the respective canyon sides or walls will rarely be found. Hence the shape of the surface of the up-stream face of the dam between elevations VI and V cannot be described as resembling that of any geometrical body. Elevation IV is coincident with another change of slope in the sides of the canyon; and the most economical mean radius corresponding to the fixed distance W_4 across the canyon at this level is found in the same manner as were found the mean radii for elevations VI and V; and the location of the center of curvature of the arch slice at this elevation is fixed, regardless of the location of the centers of the slices at elevations VI and V. In other words, the dam has no common center line, and centers are located principally with a view to getting the length of the arch as short as possible for a given distance across the canyon. The radii and thicknesses at elevations III, II and I are correspondingly determined in accordance with the procedure above outlined.

So far only the average stress **q** has been considered. Towards lower elevations where the thickness **t** of the different arch slices is considerable, it is necessary to investigate also the maximum stress to be sure that this is not above the safe limit. The maximum arch compression will be equal to

$$q \times \frac{2(R+t)}{2R+\frac{2}{2}t}$$

and will exist along the down-stream edge. The stress on the foundation does not need to be considered for dams of this type less than 200 ft. in height, as it will always be within the safe limit.

The above described method of design can also be directly applied to the multiple arch type of dam. In this case the most economical result will be obtained by taking the top width equal the distance between centers of buttresses, and the bottom width equal the distance between outside of buttresses, and by choosing the enclosed angle less than the most economical at the top where excess thickness must be provided for mechanical reasons and gradually increasing this angle towards the foundation.

THE DRAWING UP OF THE DAM

After having calculated the different radii and respective thicknesses of the several horizontal slices of the dam, the top thickness, which is generally chosen, is set off and the arcs of two concentric circles with Radii R_6 and $(R_6 + t_6)$ respectively are drawn in on the contour map; and the centers of such circles will be, of course, on a line drawn perpendicular to the center of the chord W_6. The thickness t_5 is then set off; and the arcs of the two other concentric circles, with radii R_5 and $(R_5 + t_5)$ respectively, are drawn in until intersection occurs with the contour line V—V, bearing in mind that the center common to these latter two circles does not need to be on the perpendicular to the chord W_6 above referred to. These four circles, concentric only in pairs, determine the contour of the dam between elevations VI and V. It is usually convenient and preferable to assume the down-stream faces of the upper slices of the dam adjacent the center thereof, to be vertical, at least for the first trial; although after the lower slices have been laid in, it may, at times, be found desirable to slope the central portion of such face one way or the other. The center of the arch slice at elevation V—V is correspondingly located on a perpendicular drawn through the center of the chord W_5; and the thickness t_4 is set off, assuming initially again that the central portion of the down-stream face is substantially vertical here also; although some slope may thereafter be given to it. It has been found in practice that a vertical wall at this point usually affords the most economical construction. At other points the down-stream face will always have some slope, due to the above method of design. The arcs of two concentric circles with radii R_4 and $(R_4 + t_4)$ respectively are then drawn in until intersection occurs with contour lines IV—IV; bearing in mind again that the center common to these two circles does not need to and hardly ever will lie on a center line drawn perpendicularly to the centers of either chords W_6 or W_5.

The same procedure as outlined above is thereafter followed to get the up and down-stream face lines at elevations III, II and I.

A certain amount of discretion must be exercised in locating the centers of the respective arch slices, especially where a very abrupt change occurs at one side only or upon one wall of the canyon.

While it is generally preferable to so form the slices or sectors of the dam, which lie between the several elevations, that the tops and bottoms of adjoining slices may be regarded as being superposed in strict coincidence in the manner shown in figures 4 and 5, it is desirable in some localities to face the dam with ashlar, cut-stone or the like, and it may be of advantage in such case to step the faces of the dam off as shown in the enlarged section A—B shown in figure 6. The dam will then consist of a num-

ber of cylindrical rings superposed one upon the other. The up and down stream radii in such case, however, are ascertained in exactly the same manner as in the foregoing.

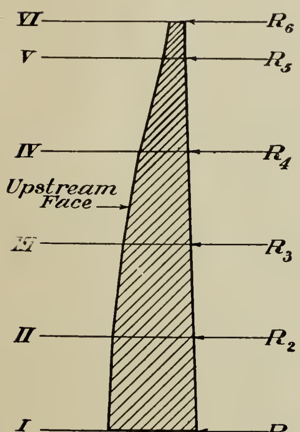


Fig. 4

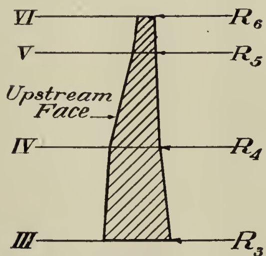


Fig. 5

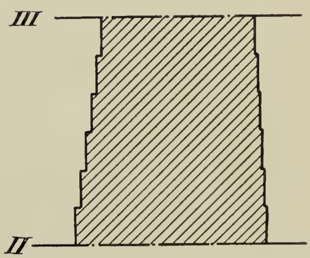


Fig. 6

As the dam must also be safe with reservoir empty, it is necessary to have t increase from the crest to the foundation to prevent overhang. The proportional increase in water pressure must, therefore, be greater than the proportional decrease in length of the up-stream radius as we proceed towards the foundation. The ratio of increase in water pressure is always fixed and the ratio of decrease in the length of the up-stream radius depends upon the slope of the canyon sides. Now, if these slopes are such that at any intermediate elevation the ratio of decrease in length of the up-stream radius (corresponding to a 133° arch) has been greater than the ratio of increase in water pressure, a decrease in the thickness of the dam at this elevation would result and the structure would be overhanging. If a certain thickness must be provided to prevent overhanging, it is most economical to throw normal load on the total area by increasing the length of the up-stream radius above that corresponding to a 133° arch for the reason that a flat arch requires less material than a more curved one of the same thickness. By flattening the arch the enclosed angle has been decreased and it will be found that to cover all practical cases the enclosed angle will have to be varied between the limits 140° and 60° in order to get the most economical dam, and one which also will satisfy the requirements for safety with no water in the reservoir.

MECHANICAL FEATURES OF STRUCTURE

Besides the great saving in material this type of dam possesses another feature of marked importance. This feature is that arch action can and will take place even very close to the foundation, for the following reasons:

When an arch dam of any type is loaded it deflects. This deflection is a maximum at the crown and is equal to

$$C \times \frac{P\,(\text{Upstream radius})^2}{E \times t}$$

where C is a factor which takes the curved beam action into consideration; E is the modulus of elasticity. For $2\theta = 90^\circ$ C equals one; for larger angles it is above one (1.15 at 133°) and for smaller angles it is below one (0.93 at 60°).

From this formula is directly seen that the deflection is practically proportional to the square of the up-stream radius divided by the thickness t. P and E are constants, so far as the comparison of the two types is concerned. The new type of dam is designed with decreasing length of up-stream radius, the old type of arch dam with constant length of radius, or if a face batter is used, with increasing length of radius from the crest towards the foundation. In the new type of arch dam the length of the up-stream radius near the foundation may be only one-third of what it is at the crest. The thickness t may be only one-half of the thickness required for the old type using the same average unit compression. The resulting deflection of the crown near the foundation will, therefore, be 2/9 or about 1/5 of what it would have been if a constant up-stream radius had been used. In other words, this new type acting as an arch is able to take up 5 times as much of the load near the foundation for the same deflection as the old type. For the small deflection in question cantilever or gravity action cannot predominate; the structure must act chiefly as an arch and as such it has the necessary strength. As the dam has weight, some gravity action will exist and in the upper sections, where the arch is long and thin, this gravity or cantilever action will prevent buckling of the arch before crushing, if the dam cross section has an area not less than one-half that of an ordinary gravity section, designed with a factor of safety against overturning of two.

The small average deflection characteristic of this new type of dam will make it possible for the structure to take care of stresses due to temperature changes and shrinkage, eliminating cracks either entirely or to a large extent. Shrinkage and low temperature both tend to shorten the length of the arch the same as the load. As the ends of the arch are fixed to the abutments, this shortening either causes cracks to develop or forces the crown back. In this

new type the average deflection being much smaller for the same amount of decrease in length of arch than in the old type, the tension necessary to cause this deflection may not exceed the ultimate tensile strength of the concrete, in which case no cracks would develop. In any event cracks are not as liable to occur as in the old type.

COMPARISON WITH EXISTING ARCH DAMS

The Bear Valley dam in Southern California, so often cited on account of its boldness, would, if reconstructed according to the foregoing method, have a factor of safety of over 12 at its weakest point, using the same cross section, or it would have a factor of safety of between 9 and 10 using the same amount of material as in the existing dam with its factor of safety of 3 as follows:

The weakest portion of the Bear Valley dam is at elevation 48 ft. below the crest; here the thickness is 8.42 ft., and, as the length of the up-stream radius is 340 ft. at this elevation, the mean compression per square inch of the masonry is found from

$$\frac{\text{Water pressure x. up-stream rad.}}{\text{area}} = \frac{48 \times 62.5 \times 340}{8.42 \times 144} = 840 \text{ lbs.},$$

which should give the dam a factor of safety of about 3. The length of the up-stream radius which should have been used in order to get the best use of the material is 160 ft. at the top instead of 335 ft. as constructed. Suppose this radius was used for the whole face, the factor of safety would thereby be increased from 3 to $\frac{340}{160} \times 3 = 6.37$ at the weakest point

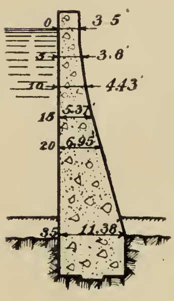
The increase in material due to the longer arch would amount to $\frac{370}{300}$, or 23% approximately, while the factor of safety has been more than doubled.

If, instead of using a constant up-stream radius of 160 ft., the length be changed so as to keep the enclosed angle as close as possible to the most economical value, a much safer structure would result. At elevation 48 ft. below the crest the length of the up-stream radius would be 80 ft. or less (the exact length cannot be given due to the absence of a contour map of the site) instead of 335 ft. as used.

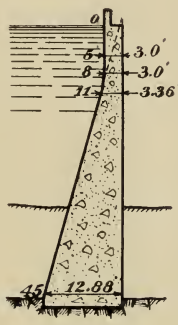
Using the same thickness of section as before, that is, 8.42 ft., the average compression per square inch at this the weakest portion of the dam can be found from

$$\frac{\text{Water pressure x up-stream radius}}{\text{area}} = \frac{48 \times 62.5 \times 80}{8.42 \times 144} = 198 \text{ lbs.}$$

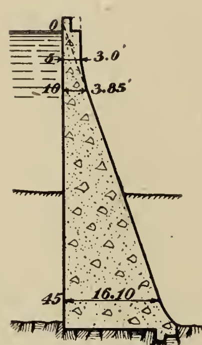
a unit compression 4¼ times smaller than the one used in the structure. The increase in material due to the longer arch will be about one-third. That is, by adding 34 per cent to the amount of material, we have raised the factor of safety from 3 to 12¾. Using the same amount of material the new type of



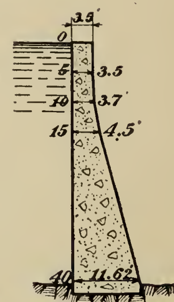
Lithgow №1



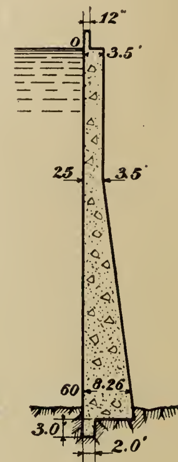
Cootamundra



Mudgee

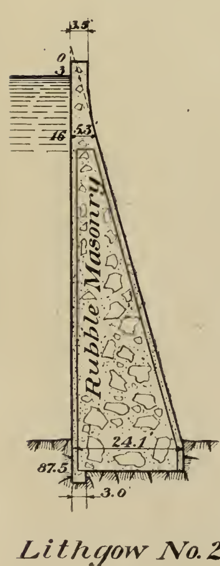


Wollongong

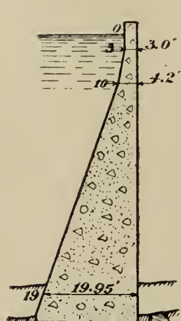


Medlow

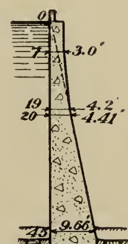
dam would show a factor of safety of 9.5 as against 3 in the dam as constructed. This goes to show that there is really absolutely no boldness about the cross section used in the Bear Valley dam. The cut and try method employed in the design, however, is certainly bold.



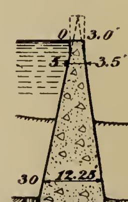
Lithgow No. 2



Tamworth



Wellington



Parkes

More recently there has been constructed a number of arched dams in New South Wales, Australia, which attract attention on account of the thinness of their sections. They have, however, been properly designed and, therefore, possess a sufficiently high factor of safety. A full description of these interesting structures can be found in a paper read by their designer, L. A. B. Wade, M. I. C. E., before the Institution of Civil Engineers, and from which the following table and cross sections are copied.

Table showing Details of Curved Masonry Dams built by the Public Works Department of New South Wales.

Locality	Max. ht. above foundations	Total Length	Top thickness.	Depth below crest of top thickness.	Thickness of base	Radius of curved part	Limit of pressure in tons *	Character of rock forming site and used in construction	Date of construction
	ft.	ft.	ft.	ft.	ft.	ft.	per sq. ft.		
Lithgow No. 1	35.0	178	3.5	3.5	10.88	100	11.2	Sandstone	1896
Parkes	33.5	540	3.0	6.0	13.5	300	26.9	Granite	1897
Cootamundra	46.0	640	3.0	8.0	13.0	250	28.0	Granite	1898
Tamworth	61.0	440	3.0	3.0	21.5	250	22.4	Granite	1898
Wellington	48.0	350	3.0	7.0	10.0	150	22.4	Conglomerate	1899
Mudgee	50.0	498	3.0	5.0	18.0	253	22.4	Altered slate	1899
Wollongong	42.0	535	3.5	5.0	11.62	200	22.4	Basalt	.....
Lithgow No. 2	87.0	221	3.0	3.0	24.0	100	11.2	Sandstone	1906
Medlow	65.0	124	3.5	21.0	8.96	60	13.4	Sandstone	1906

* Short tons of 2000 lbs.

The maximum stresses used are somewhat higher than usual, although only half of what they are in the Bear Valley dam. Using the new type described above for these sites the maximum stresses could be brought down to about one-half of their present values, resulting in still safer structures with equal amounts of material. To use the same stresses and, therefore, a thinner section requiring less material would in these cases perhaps not be advisable as the original sections are already rather thin.

The main features embodied in this new type of dam have been patented and the rights to the design are owned by **F. G. BAUM & COMPANY**, Chronicle Building, San Francisco, Cal., who are prepared to furnish complete designs and estimates, and supervise the construction. The charges are no higher than for ordinary engineering work. Should the owner prefer to have his dam designed and built by his own force, this company solicits correspondence relative to the use of this design.